

Self-test for prospective MSc students

Congratulations on your MSc offer and welcome to our program!

We would like you to arrive at Sussex well-prepared for your course and to that end we have prepared this self-test in order to help you check your readiness and identify any areas that might need revision or further study. These problems are not meant to be extensive and covering

$$\frac{dy}{dx} \quad 2xy = x$$

$$\frac{d^2y}{dt^2} \quad \frac{dy}{dt} \quad 30y = e^{6t}$$

Fitting data points:

Fit a straight line and a quadratic to the data in the Table using any numerical method you like (e.g. if you know Python you can use `curve_fit` function from `scipy.optimize`). Which is better? (Hint: Compare the standard deviations.)

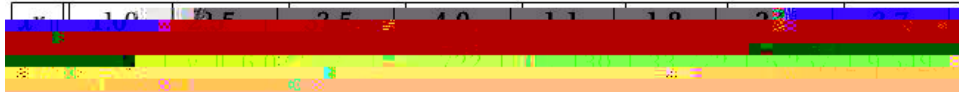


Figure 1: Data for fitting.

Diffraction of light: In optics, you have learned that light bends around objects, i.e. exhibits diffraction. One of the simplest cases to study is the bending of light around a straight edge. In this case, we find that the intensity of light varies as we move away from the edge according to:

$$I = I_0 [C(v) + 0.5]^2 + [S(v) + 0.5]^2$$

where I_0 is the intensity of the incident light, v is proportional to the distance travelled, and $C(v)$ and $S(v)$ are the Fresnel integrals:

$$C(v) = \int_0^v \cos(w^2) dw$$

and

$$S(v) = \int_0^v \sin(w^2) dw$$

Using any method, numerically integrate the Fresnel integrals, and thus evaluate $I = I_0$ as a function of v for $0 \leq v \leq 10$. Plot your results for C ; S and $I = I_0$. Do they agree with what you have learned about diffraction in Optics? As an extra twist, try to this *computationally efficiently*.

Challenge: Derive the formula for I .

Physics:

Mechanics: Two particles move about each other in circular orbits under the influence of their mutual gravitational force, with a period T . At some time $t = 0$, they are suddenly stopped and then they are released and allowed to fall into each other. Find the time T after which they collide, in terms of T .

Mechanics: Small oscillations: A particle moves under the influence of the potential $V(x) = Cx^n e^{-ax}$. Find the frequency of small

If the initial mass is M , and the initial v is zero, integrate the above equation to obtain

$$m = M \frac{1}{\sqrt{1 - v^2/c^2}}$$

Relativity: Colliding photons: Two photons each have energy E . They collide at an angle θ and create a particle of mass M . What is M ?

Electrodynamics: Consider an electrostatic potential given by

$$V = \frac{q}{4\pi\epsilon_0 r} (1 + br^a)$$

where a and b are constants.

- Find the distribution of charge (both continuous and discrete) that will give this potential.
- What, if anything, is special when $a = 2b$?
- Interpret your results physically.

Electrodynamics/Waves: A transverse plane wave impinges normally in vacuum on a perfectly absorbing flat screen.

- From the law of conservation of linear momentum, show that the pressure (radiation pressure) exerted on the screen is equal to the field energy per unit volume in the wave.
- Let the incident radiation have a flux of 1.4 kW/m^2 . The absorbing screen has a mass of 1 g/m^2 . What is the screen's acceleration due to radiation pressure?

Electrodynamics/Waves: A monochromatic plane wave has complex electric field $E(r; t) = E_0 \exp[i(k \cdot r - \omega t)]$. It travels in the $+z$ direction in a lossless isotropic medium with relative permittivity $\epsilon_r = 4$ and relative permeability $\mu_r = 1$. The field is linearly polarized in the x -direction, with frequency $\omega = 1 \text{ GHz}$ and a peak field $+10^3 \text{ V/m}$ at $t = 5 \text{ ns}$ and $z = 1 \text{ m}$.

- Define the medium refractive index and find the angular frequency, phase velocity, wavenumber, wave vector, and wavelength.
- Obtain the real instantaneous expression for $E(r; t)$ valid for any position and time.

- (c) Obtain the real instantaneous expression for the magnetic field $B(r; t)$