# Self-test for prospective MSc students

Congratulations on your MSc o er and welcome to our program! We would like you to arrive at Sussex well-prepared for your course and to that end we have prepared this self-test in order to help you check your readiness and identify any areas that might need revision or further study. These problems are not meant to be extensive and covering

$$\frac{dy}{dx} \quad 2xy = x$$

$$\frac{d^2y}{dt^2} \quad \frac{dy}{dt} \quad 30y = e^{6t}$$

Fitting data points:

Fit a straight line and a quadratic to the data in the Table using any numerical method you like (e.g. if you know Python you can use `curve_fit` function from `scipy.optimize`). Which t is better? (Hint: Compare the standard deviations.)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | 1.0 | 2.5 | 3.5 | 4.0 | 1.1 | 1.8 | [illegible] | 3.7 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

Figure 1: Data for tting.

Di raction of light: In optics, you have learned that light bends around objects, i.e. exhibits di raction. One of the simplest cases to study is the bending of light around a straight edge. In this case, we nd that the intensity of light varies as we move away from the edge according to:

$$I = 0{:}5I_0 f[C(v) + 0{:}5]^2 + [S(v) + 0{:}5]^2 g$$

where $I_0$ is the intensity of the incident light, $v$ is proportional to the distance travelled, and $C(v)$ and $S(v)$ are the Fresnel integrals:

$$C(v) = \int_0^v \cos( w^2 = 2)dw$$

and

$$S(v) = \int_0^v \sin( w^2 = 2)dw$$

Using any method, numerically integrate the Fresnel integrals, and thus evaluate $I=I_0$ as a function of $v$ for $0 \quad v \quad 10$. Plot your results for $C; S$ and $I=I_0$. Do they agree with what you have learned about di raction in Optics? As an extra twist, try to this *computationally e ciently*.

Challenge: Derive the formula for $I$.

## Physics:

Mechanics: Two particles move about each other in circular orbits under the in uence of their mutual gravitational force, with a period . At some time $t = 0$, they are suddenly stopped and then they are released and allowed to fall into each other. Find the time $T$ after which they collide, in terms of .

Mechanics: Small oscillations: A particle moves under the in uence of the potential $V(x) = Cx^n e^{ax}$. Find the frequency of small

If the initial mass is $M$, and the initial $v$ is zero, integrate the above equation to obtain

$$m = M\sqrt{\frac{1 \quad v}{1+v}}:$$

Relativity: Colliding photons: Two photons each have energy $E$. They collide at an angle and create a particle of mass $M$. What is $M$?

Electrodynamics: Consider an electrostatic potential given by

$$= \frac{q}{4 \quad _0}\frac{e^{\quad ar}}{r}(1+br)$$

where $a$ and $b$ are constants.

(a) Find the distribution of charge (both continuous and discrete) that will give this potential.

(b) What, if anything, is special when $a = 2b$?

(c) Interpret your results physically.

Electrodynamics/Waves: A transverse plane wave impinges normally in vacuum on a perfectly absorbing at screen.

(a) From the law of conservation of linear momentum, show that the pressure radiation pressure exerted on the screen is equal to the eld energy per unit volume in the wave.

(b) Let the incident radiation have a ux of $1{:}4\,kW{=}m^2$. The absorbing screen has a mass of $1\,g{=}m^2$. What is the screens acceleration due to radiation pressure?

Electrodynamics/Waves: A monochromatic plane wave has complex electric eld $E(r;t) = E_0 \exp[i(k \quad r \quad !t)]$. It travels in the $+\ z$ direction in a lossless isotropic medium with relative permittivity $_r = 4$ and relative permeability $_r \quad 1$. The eld is linearly polarized in the x-direction, with frequency $= 1\,GHz$ and a peak eld $+10^{3}\,V{=}m$ at $t = 5\,ns$ and $z = 1\,m$.

(a) De ne the medium refractive index and nd the angular frequency, phase velocity, wavenumber, wave vector, and wavelength.

(b) Obtain the real instanteneous expression for $E(r;t)$ valid for any position and time.

(c) Obtain the real instanteneous expression for the magnetic eld $B(r;tB$